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Spacecraft Orbit Determination with The B-spline Approximation Method*

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Abstract It is known that the dynamical orbit determination is the most common way to get the precise orbits of spacecraft. However, it is hard to build up the precise dynamical model of spacecraft sometimes. In order to solve this problem, the technique of the orbit determination with the B-spline approximation method based on the theory of function approximation is presented in this article. In order to verify the effectiveness of this method, simulative orbit determinations in the cases of LEO (Low Earth Orbit), MEO (Medium Earth Orbit), and HEO (Highly Eccentric Orbit) satellites are performed, and it is shown that this method has a reliable accuracy and stable solution. The approach can be performed in both the conventional celestial coordinate system and the conventional terrestrial coordinate system. The spacecraft's position and velocity can be calculated directly with the B-spline approximation method, it needs not to integrate the dynamical equations, nor to calculate the state transfer matrix, thus the burden of calculations in the orbit determination is reduced substantially relative to the dynamical orbit determination method. The technique not only has a certain theoretical significance, but also can serve as a conventional algorithm in the spacecraft orbit determination.

Key words: spacecraft—celestial mechanics: orbit calculation and determination—methods: numerical

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1. INTRODUCTION

In the spacecraft orbit determination, the state equations and measurement equations can be described with the system as follows:

$$\begin{cases} \dot{\boldsymbol{X}} = \boldsymbol{F}(\boldsymbol{X}, t) \\ \boldsymbol{Y}_i = \boldsymbol{G}(\boldsymbol{X}_i, \boldsymbol{t}_i) + \boldsymbol{\varepsilon}_i \end{cases}, \tag{1}$$

in which $\boldsymbol{X}$ indicates the quantities of spacecraft orbit state, $\boldsymbol{Y}$, measured quantities, $\boldsymbol{\varepsilon}$, measuring noises. The above system is often linearized in the orbit determination, as the linear estimation is theoretically ripe enough. In the orbit determination, it is essential to calculate those two matrices, of which one is $\frac{\partial \boldsymbol{G}}{\partial \boldsymbol{X}}$, the matrix of partial derivatives of measured quantities relative to state quantities, another is $\frac{\partial \boldsymbol{X}}{\partial \boldsymbol{X}_0}$, the state transfer matrix. In general, to obtain the state transfer matrix, a numerical integration is made to the variational equations as follows[1]3−5[2−5]

$$\begin{cases} \dot{\boldsymbol{\Phi}}(t, t_0) = \frac{\partial \boldsymbol{F}(\boldsymbol{X}(t), t)}{\partial \boldsymbol{X}(t)} \boldsymbol{\Phi}(t, t_0) \\ \boldsymbol{\Phi}(t_0, t_0) = \boldsymbol{I}_n \end{cases}, \tag{2}$$

in which t_0 is initial epoch, $\dot{\boldsymbol{\Phi}}$, state transfer matrix, $\boldsymbol{I}$, unit matrix. If the spacecraft is a satellite, and its orbital arc segment to be determined is not too long, then the approximate analytical equations may be used, which have been thoroughly discussed in the articles [4] and [6]. No matter what approximation approach is applied to the calculation of state transfer matrix, can it be converged, only when the mapping (here it is the product of the partial derivative matrix of measured quantities relative to the state quantities with the state transfer matrix) was compressed one.

In the process of orbit determination, it is generally unavoidable to integrate the state quantities and to calculate the state transfer matrix. Although the calculation of state transfer matrix may be simplified in certain cases, such as by the difference method or the analytical approximation as mentioned above, however, the integrated results (either analytical or numerical ones) are substantially utilized. The necessary conditions for the orbit determination with the dynamical batch-processing method are two folds: the spacecraft orbit dynamics can be relatively precisely modeled, and the numerical integration is fairly reliable (the latter may be safely realized in a certain arc segment thank of the development of modern computation methods). However, if one of above demands conflicted seriously with the reality, the orbit determination will go to failure.

In the practice of radio exterior trajectory measurement to determine the motion state of a spacecraft, it often happens that the main acting force on the spacecraft is ignorant or inaccurately known. This being the case, it will be impossible to determine dynamically an orbit. As an example, in the landing phase of a deep space probe, its sensor is required to measure frequently the information of landing surface and feedback the signals to perform the orbit manoeuvre. This arc segment is none other than the key point in the measurement and control, and the determination of the probe's motion state is extremely important, so it is necessary to study deeply the corresponding method. As a method used before, around an orbit manoeuvre the orbit determination was dynamical; during the orbit manoeuvre the orbit was fitted with a polynomial[7]; and finally they were put together. It is well known that the polynomial fitting is good for a short arc, as far as a long arc is concerned, however, it can not come to an efficient approximation.

For the differential equations describing the orbital dynamics of an ordinary spacecraft, if its force function $\boldsymbol{F}(\boldsymbol{X}, t)$ is of C^1 in a rectangular region Γ (for example when a spacecraft gets in and out of the Earth shadow), and $\boldsymbol{F}(\boldsymbol{X}, t)$ satisfies the Lipschitz condition with $\boldsymbol{X}$ as its state quantities, then the solution of the differential equations exists and it is unique[8−9]. However, Joseph Liouville has demonstrated that explicit solutions could not be obtained for many differential equations, namely, it is impossible to give an explicit expression $\boldsymbol{X}_t = \boldsymbol{\phi}(\boldsymbol{X}_0, t_0, t, \boldsymbol{\mu})$, in which $\boldsymbol{\mu}$ being a parameter vector.

The solution of the differential equations forms a mapping g^t from the phase space to oneself: $R^6 \rightarrow R^6$, and the mapping g^t forms a differentially homeomorphic one-parameter group in the phase space. This mapping group g^t is also known as a phase flow along with the dynamical equations of the spacecraft. Although the explicit expression of equation solution can not be got, the solution does exist. Therefore, it is possible to approximate directly the solution of differential equations in terms of the theory of function approximation, instead of dynamical analysis. This is the basic idea of this article.

2. B SPLINE APPROXIMATION METHOD FOR ORBIT DETERMINATION

2.1 B Spline Function

In recent decades, for both theoretical research and practical use the theory of function approximation has obtained a significant development, in which the study of spline function is a very active branch. In the spline theory, B spline is renowned for its graceful theory and typicality in numerical calculation, and it has become the essential tool in the computer-aided geometric design, being widely applied to the configuration designs of vehicles, ships and aircrafts etc. B spline has provided an unified mathematical method for free curves and curved surfaces.

In the early stage, the application of the spline approximation method in the measurement and control system of space flight might arise from some target-range experiments, while the missile trajectory would be approximated with the spline approximation in the postprocessing of measured data. Researchers in this country have applied this theory to engineering already[10].

The B-spline base may be defined in many ways, and an iterative definition is given here. For the given division $U : \{u_i\}_{i=-\infty}^{+\infty} (u_i \leq u_{i+1}, i = 0, \pm 1, \cdots)$ on the parameter u axis, the function $N_{i,p}(u)$ defined by the following iterations is known as the p-degree (or $(p+1)$-order) B-spline base corresponding to the division U

$$\begin{cases} N_{i,p}(u) = \begin{cases} 1, u \in [u_i, u_{i+1}) \\ 0, \text{otherwise} \end{cases} \\ N_{i,p}(u) = \frac{u-u_i}{u_{i+p}-u_i} N_{i,p-1}(u) + \frac{u_{i+p+1}-u}{u_{i+p+1}-u_{i+1}} N_{i+1,p-1}(u), p \geq 2 \end{cases}, \tag{3}$$

in which $[u_i, u_{i+1})$ is the nodal domain, and its length may be zero, namely, the nodal points are allowed to be repeated. If $\frac{0}{0}$ should appear in above equations, $\frac{0}{0} = 0$ will be stipulated. For the spline with equally spaced nodal points, the definition may also be as follows [11]315−331. Let us denote

$$u_+^m = \begin{cases} u^m, (u \geq 0) \\ 0, (u < 0) \end{cases} , \tag{4}$$

which is known as m-degree semi-length monomial, in which $m = 1, 2, \cdots$, and it is stipulated that

$$u_+^0 = \begin{cases} 1, (u > 0) \\ \frac{1}{2}, (u = 0) \\ 0, (u < 0) \end{cases} . \tag{5}$$

Let B_k is a function in $(-\infty, +\infty)$

$$B_k = \frac{\sum\limits_{j=0}^{k+1} (-1)^j \begin{pmatrix} k+1 \\ j \end{pmatrix} \left(x + \frac{k+1}{2} - j\right)_+^k}{k!} \tag{6}$$

which is known as the k-degree base spline or the B spline. If

$$B_3 = \begin{cases} 0, & |x| \geq 2 \\ \frac{1}{2}|x|^3 - x^2 + \frac{2}{3}, & |x| \leq 1 \\ -\frac{1}{6}|x|^3 + x^2 - 2|x| + \frac{4}{3}, & 1 < |x| < 2 \end{cases} , \tag{7}$$

$B(\tau)$ is called the 3-order standard B-spline. If the region on which the sample data are treated is $[T_2, T_{P-1}]$, let's denote

$$\begin{cases} h = \frac{(T_{P-1} - T_2)}{P-3} \\ s(t) = \sum\limits_{j=1}^{P} b_j B\left(\frac{t - T_j}{h}\right) \end{cases} , T_j = T_2 + (j-2)\, h, \tag{8}$$

in which P represents the number of sections of the region, h, the length of the region, then

$$\begin{cases} T_1 = T_2 - h \\ T_P = T_2 + (P-2)\, h = T_{P-1} + h \end{cases} . \tag{9}$$

In practical applications, it is possible to choice some extended function approximation methods of B-splines, such as the inhomogeneous rational B-spline, etc. In the articles [12] and [13], some highly efficient algorithms of spline base have been given, and the spline theory has been thoroughly analyzed.

2.2 B-spline Approximation Method of Orbit Determination

Here, instead of the general form of approximation, we will discuss only the B-spline method adopted in this article, which may be naturally extended to the orbit determination of other forms of approximation.

Adopting the B-spline approximation method, the form of approximation of spacecraft's state is

$$\begin{cases} x(t) = \sum\limits_{j=1}^{P} \alpha_j B\left(\frac{t-T_j}{h}\right) \\ y(t) = \sum\limits_{j=1}^{P} \beta_j B\left(\frac{t-T_j}{h}\right) \\ z(t) = \sum\limits_{j=1}^{P} \gamma_j B\left(\frac{t-T_j}{h}\right) \end{cases}, \begin{cases} h = \frac{T_{P-1}-T_2}{P-3} \\ T_j = T_2 + (j-2)\,h \end{cases}, \tag{10}$$

and the linearized equation of orbit determination is

$$\boldsymbol{O} - \boldsymbol{C} = \left[\frac{\partial \boldsymbol{G}}{\partial \boldsymbol{X}}\right] \begin{bmatrix} \boldsymbol{\Xi} \\ \boldsymbol{\Theta} \end{bmatrix} \Delta\boldsymbol{\psi}, \tag{11}$$

in which $\boldsymbol{\psi} = [\alpha_1, \cdots, \alpha_P, \beta_1, \cdots, \beta_P, \gamma_1, \cdots, \gamma_P]^{\mathrm{T}}$ are parameters to be estimated, $\Delta\boldsymbol{\psi}$ is the correction. $\boldsymbol{\Xi}$ is the linear mapping matrix from the parameters to the position vector, $\boldsymbol{O}$, measured quantities, $\boldsymbol{C}$, theoretical values of the measured quantities, $\boldsymbol{\Theta}$, the linear mapping matrix from the parameters to the velocity vector. It can be seen that all these are directly of linear problem, not nonlinear one.

Further more, for the equation of orbit determination (11), its nonlinearity is caused only by the nonlinear relationship between the measured quantities and the state ones. If the measured quantities were a linear combination of the state quantities, the entire process of orbit determination would be linear, and the iterative solution is unnecessary. The specific forms of matrices are given below, they are very concise. The following vectors are denoted as

$$\begin{cases} \boldsymbol{B}_1 = \left[\, B\left(\frac{t_1-T_1}{h}\right) \; B\left(\frac{t_1-T_2}{h}\right) \; \cdots \; B\left(\frac{t_1-T_P}{h}\right) \,\right] \\ \boldsymbol{B}_2 = \left[\, B\left(\frac{t_2-T_1}{h}\right) \; B\left(\frac{t_2-T_2}{h}\right) \; \cdots \; B\left(\frac{t_2-T_P}{h}\right) \,\right] \\ \vdots \\ \boldsymbol{B}_m = \left[\, B\left(\frac{t_m-T_1}{h}\right) \; B\left(\frac{t_m-T_2}{h}\right) \; \cdots \; B\left(\frac{t_m-T_P}{h}\right) \,\right] \end{cases}, \tag{12}$$

then

$$\boldsymbol{\Xi} = \begin{bmatrix} \boldsymbol{B}_1 & \boldsymbol{B}_1 & \boldsymbol{B}_1 \\ \boldsymbol{B}_2 & \boldsymbol{B}_2 & \boldsymbol{B}_2 \\ \vdots & \vdots & \vdots \\ \boldsymbol{B}_m & \boldsymbol{B}_m & \boldsymbol{B}_m \end{bmatrix}. \tag{13}$$

In some cases, the data may be a function of velocity, such as the instantaneous Doppler data. In this case, it is not appropriate to calculate the theoretical value of velocity by means of numerical differentiation, instead of spline functions.

As for the B spline mentioned above, the velocity components are

$$\begin{cases} \dot{x}(t) = \sum\limits_{j=1}^{P} \frac{\alpha_j \dot{B}\left(\frac{t-T_j}{h}\right)}{h} \\ \dot{y}(t) = \sum\limits_{j=1}^{P} \frac{\beta_j \dot{B}\left(\frac{t-T_j}{h}\right)}{h} \\ \dot{z}(t) = \sum\limits_{j=1}^{P} \frac{\gamma_j \dot{B}\left(\frac{t-T_j}{h}\right)}{h} \end{cases}, \tag{14}$$

in which

$$\dot{B} = \begin{cases} 0, & |x| \geq 2 \\ \frac{3}{2}|x|^2 \times \kappa - 2x, & |x| < 1 \\ -\frac{1}{2}|x|^2 \times \kappa + 2x - 2 \times \kappa, & 1 \leq |x| < 2 \end{cases} ,$$

$$\kappa = \mathrm{sign}\,(x) = \begin{cases} 1, & x \geq 0 \\ -1, & x < 0 \end{cases} .$$

If we denote

$$\begin{cases} \dot{\boldsymbol{B}}_1 = \left[\ \dot{B}\left(\frac{t_1 - T_1}{h}\right) \ \ \dot{B}\left(\frac{t_1 - T_2}{h}\right) \ \cdots \ \dot{B}\left(\frac{t_1 - T_P}{h}\right)\ \right] \\ \dot{\boldsymbol{B}}_2 = \left[\ \dot{B}\left(\frac{t_2 - T_1}{h}\right) \ \ \dot{B}\left(\frac{t_2 - T_2}{h}\right) \ \cdots \ \dot{B}\left(\frac{t_2 - T_P}{h}\right)\ \right] \\ \vdots \\ \dot{\boldsymbol{B}}_m = \left[\ \dot{B}\left(\frac{t_m - T_1}{h}\right) \ \ \dot{B}\left(\frac{t_m - T_2}{h}\right) \ \cdots \ \dot{B}\left(\frac{t_m - T_P}{h}\right)\ \right] \end{cases} , \tag{15}$$

we shall have the mapping matrix from the parameters to the velocity vector (It may be understood as a partial derivative matrix of the velocity over the parameters, and in fact they are linearly related to each other) to be

$$\boldsymbol{\Theta} = \frac{1}{h} \begin{bmatrix} \dot{\boldsymbol{B}}_1 & \dot{\boldsymbol{B}}_1 & \dot{\boldsymbol{B}}_1 \\ \dot{\boldsymbol{B}}_2 & \dot{\boldsymbol{B}}_2 & \dot{\boldsymbol{B}}_2 \\ \vdots & \vdots & \vdots \\ \dot{\boldsymbol{B}}_m & \dot{\boldsymbol{B}}_m & \dot{\boldsymbol{B}}_m \end{bmatrix} . \tag{16}$$

Up to now, a complete series of equations have been given for the orbit determination with the B-spline approximation method.

2.3 Orbit Estimation with Dynamical Constraints

As mentioned above, an orbit determination method with the B spline approximation has been given. If the B-spline approximation method, however, is merely in use, the dynamical information of the orbit will be neglected, and in a certain sense this will be a loss of information. In the case of possession of less measured data, the solution would be unsatisfactory. In this case, some constraints may be applied to the orbit in terms of spacecraft dynamics.

If at the moment t_k the spacecraft is in conformity with the dynamical equation

$$\ddot{\boldsymbol{r}}_k = \boldsymbol{f}\left(\boldsymbol{X}_k, t_k\right) , \tag{17}$$

in which $\boldsymbol{r}$ is the position of the spacecraft, denoting

$$\ddot{\boldsymbol{B}}_k = \left[\ \ddot{B}\left(\frac{t_k - T_1}{h}\right) \ \ \ddot{B}\left(\frac{t_k - T_2}{h}\right) \ \cdots \ \ddot{B}\left(\frac{t_k - T_P}{h}\right)\ \right] , \tag{18}$$

in which $\ddot{B}(x)$ may be got from the 1-order derivative of $\dot{B}(x)$, and hereby, if a matrix is defined as

$$\boldsymbol{H}_k = \frac{1}{h^2} \begin{bmatrix} \ddot{\boldsymbol{B}}_k & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \ddot{\boldsymbol{B}}_k & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \ddot{\boldsymbol{B}}_k \end{bmatrix} , \tag{19}$$

then, the dynamical equation of spacecraft may be written as

$$\frac{1}{h^2}\begin{bmatrix} \ddot{\boldsymbol{B}}_k & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \ddot{\boldsymbol{B}}_k & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \ddot{\boldsymbol{B}}_k \end{bmatrix}\boldsymbol{\psi} = \begin{bmatrix} \boldsymbol{f}_x(\boldsymbol{X}_k, t_k) \\ \boldsymbol{f}_y(\boldsymbol{X}_k, t_k) \\ \boldsymbol{f}_z(\boldsymbol{X}_k, t_k) \end{bmatrix}, \tag{20}$$

namely,

$$\boldsymbol{H}_k\boldsymbol{\psi} = \boldsymbol{f}_k(\boldsymbol{X}_k, t_k). \tag{21}$$

Thus, the dynamical constraint on the spacecraft orbit is transformed from the differential equation into the algebraic one. In the equation (21), because the force is a nonlinear function of state quantities, this equation remains nonlinear. However, while solving the parameters, an iterative process is often needed owing to the nonlinear measurement equations. For this reason, the above equation may be iterated in synchronization with the measurement equations, namely, while estimating the parameters of equation, it will be sufficient to take the left side of the equation as linear, and to give the force function in the right side by the parameters estimated from the very time of iteration.

Denoting

$$\boldsymbol{H} = \begin{bmatrix} \boldsymbol{H}_1 \\ \boldsymbol{H}_2 \\ \vdots \end{bmatrix}, \boldsymbol{f} = \begin{bmatrix} \boldsymbol{f}_1 \\ \boldsymbol{f}_2 \\ \vdots \end{bmatrix}, \tag{22}$$

now the least square solution under the constraint condition is required

$$\min \left\| \boldsymbol{O} - \boldsymbol{C} - \left[\frac{\partial \boldsymbol{G}}{\partial \boldsymbol{X}}\right]\begin{bmatrix} \boldsymbol{\Xi} \\ \boldsymbol{\Theta} \end{bmatrix}\Delta\boldsymbol{\psi} \right\|^2_{\boldsymbol{H}\boldsymbol{\psi} = \boldsymbol{f}}.$$

It is suggested to transform the constrained optimization problem into a non-constrained one by the Lagrange method of multipliers, and the conclusion is given here with the deduction omitted:

$$\begin{bmatrix} \left[\left[\frac{\partial \boldsymbol{G}}{\partial \boldsymbol{X}}\right]\begin{bmatrix} \boldsymbol{\Xi} \\ \boldsymbol{\Theta} \end{bmatrix}\right]^{\mathrm{T}}\left[\frac{\partial \boldsymbol{G}}{\partial \boldsymbol{X}}\right]\begin{bmatrix} \boldsymbol{\Xi} \\ \boldsymbol{\Theta} \end{bmatrix} & \boldsymbol{H}^{\mathrm{T}} \\ \boldsymbol{H} & \boldsymbol{0} \end{bmatrix}\begin{bmatrix} \Delta\boldsymbol{\psi} \\ \boldsymbol{\lambda} \end{bmatrix} = \begin{bmatrix} \left[\left[\frac{\partial \boldsymbol{G}}{\partial \boldsymbol{X}}\right]\begin{bmatrix} \boldsymbol{\Xi} \\ \boldsymbol{\Theta} \end{bmatrix}\right]^{\mathrm{T}}[\boldsymbol{O} - \boldsymbol{C}] \\ \boldsymbol{f} \end{bmatrix}, \tag{23}$$

in which $\boldsymbol{\lambda}$ is a constraint multiplier vector. As this equation has been a properly posed one with its coefficients being symmetrical and positive definite, it may be solved by use of Cholesky decomposition method [11]36−40.

In the above process, it has been recognized tacitly that the dynamical modeling of the constraint conditions of the spacecraft in the given time is precise. It may be found that in this process, as the number of constraints increases, the dimensional number of the equation to be solved becomes greater, which makes the calculation have to meet more demands. Another train of thought may be adopted here: by admitting that there exist some errors in the dynamical modeling of the spacecraft (really it's a fact), the constraint errors are evaluated, and finally adjusted together with the observational data. Here with the deduction omitted, the final adjustment equation is directly given as

$$\begin{aligned}&\left\{\sum_{l=1}\left[\left[\frac{\partial \boldsymbol{G}}{\partial \boldsymbol{X}}\right]_l\left[\begin{array}{c}\boldsymbol{\Xi}\\ \boldsymbol{\Theta}\end{array}\right]_l\right]^{\mathrm{T}}\boldsymbol{R}_l^{-1}\left[\left[\frac{\partial \boldsymbol{G}}{\partial \boldsymbol{X}}\right]_l\left[\begin{array}{c}\boldsymbol{\Xi}\\ \boldsymbol{\Theta}\end{array}\right]_l\right]+\sum_{k=1}\boldsymbol{H}_k^{\mathrm{T}}\boldsymbol{Q}_k^{-1}\boldsymbol{H}_k^{\mathrm{T}}\right\}\Delta\boldsymbol{\psi}\\ &=\sum_{l=1}\left[\left[\frac{\partial \boldsymbol{G}}{\partial \boldsymbol{X}}\right]_l\left[\begin{array}{c}\boldsymbol{\Xi}\\ \boldsymbol{\Theta}\end{array}\right]_l\right]^{\mathrm{T}}\boldsymbol{R}_l^{-1}\left[\boldsymbol{O}_l-\boldsymbol{C}_l\right]+\sum_{k=1}\boldsymbol{H}_k^{\mathrm{T}}\boldsymbol{Q}_k^{-1}\boldsymbol{f}_k\,,\end{aligned}\qquad(24)$$

in which $\boldsymbol{R}_l^{-1}$ is the covariance matrix of the measured data, $\boldsymbol{Q}_k^{-1}$, that of the orbit noises under constraints. As this train of thought goes, the dimension of equation is the same as the case without constraints, the amount of calculation reduces, and the programming becomes more convenient.

It is worth noting that while solving the equation (24), the initial estimates of the parameters should be obtained beforehand with the equation (11), and then the adjustment is performed with the equation (24). While the dynamical constraints of the orbit are applied, the demands upon the dynamical conditions are lower than those of the orbit determination with dynamical method. In the latter case, it is required that the dynamical state should be definite in the entire arc segment, but in this case, only in a part of time the dynamical state is required to be known.

3. CALCULATION TEST

3.1 Example of Orbit Determination of A LEO Satellite

In order to test the efficiency of the B-spline approximation method of orbit determination, orbit calculations for the LEO satellite, MEO satellite and HEO satellite are carried out in this article, respectively.

The initial Kepler orbital elements of the LEO satellite at the epoch 2012-07-01 12:00:00 are: $a = 7717.377350$ km, $e = 0.001285$, $i = 62.336°$, $\Omega = 285.635°$, $\omega = 123.601°$, $M = 205.151°$.

For the orbit calculation, the KSG (Krogh-Shampine-Gordon) integrator is adopted, which, through directly solving a 2-order differential equation set, is an improved type of the Adams-Cowell integrator, and is one of the most precise integrators applied in the present man-made satellite orbit calculation. The articles [1]246-250 and [6] have thoroughly described the integrator structure.

The perturbing forces to be considered in the orbit extrapolation are: (1) the gravitational perturbation of the non-spherical Earth's shape, (2) the atmospheric perturbation, (3) the mass-point gravitational perturbations from the sun and the moon, (4) the perturbation of light pressure, (5) the perturbation of rigid and ocean tides, (6) the effect of general relativity.

For the sake of simplicity, the measured quantities are taken as X, Y and Z; data sampling interval for each datum, 2 s; measuring noise, a Gaussian distribution with the standard error of 10 cm. When $P = 30$, the orbit determination result is shown in comparison with a nominal orbit in Fig.1. R, T and N indicate the radial, tangential and normal directions, respectively. To compare the approximation performance of the B splines with different nodal point numbers, the orbit determination result when $P = 20$ is shown in Fig.2.

3.2 Example of Orbit Determination of A MEO Satellite

The initial Kepler orbital elements of a MEO satellite at the epoch 2012-07-01 12:00:00 are: $a = 26781.970665$ km, $e = 0.00725$, $i = 62.336°$, $\Omega = 207.672°$, $\omega = 180.658°$, $M = 316.319°$. The perturbing forces to be considered in the orbit extrapolation are: (1) the gravitational perturbation of non-spherical Earth's shape, (2) the perturbation of light pressure, (3) the gravitational perturbations from the sun and the moon, (4) the perturbation of rigid and ocean tides, (5) the effect of general relativity.

The sampling interval and noise of the measured data of the experimental MEO satellite are the same as those of low-orbit satellite. When $P = 30$, the orbit determination result is shown in Fig.3. The orbit determination result when $P = 20$ is given in Fig.4. It may be made out that for a MEO satellite the high-frequency signals of orbit perturbation are weaker than those of a low-orbit satellite, and the periodic terms are not so distinct as the low-orbit satellite owing to the reduced magnitude of the gravitational perturbation of the non-spherical Earth's shape. Theoretically, the orbit approximation should be better when the number of nodal points is increased, but in practical applications, it is necessary to take account of the various factors, such as whether the measured data and the amplitude of orbit variation are in balance. In our calculation, as an example, for the low-orbit satellite, the approximation effect with more nodal points is better than that with less nodal points; but for the MEO satellite, the approximation performance may not be raised by increasing nodal points due to the smaller amplitude of high-frequency signals caused by the orbit variation as compared with the low-orbit satellite.

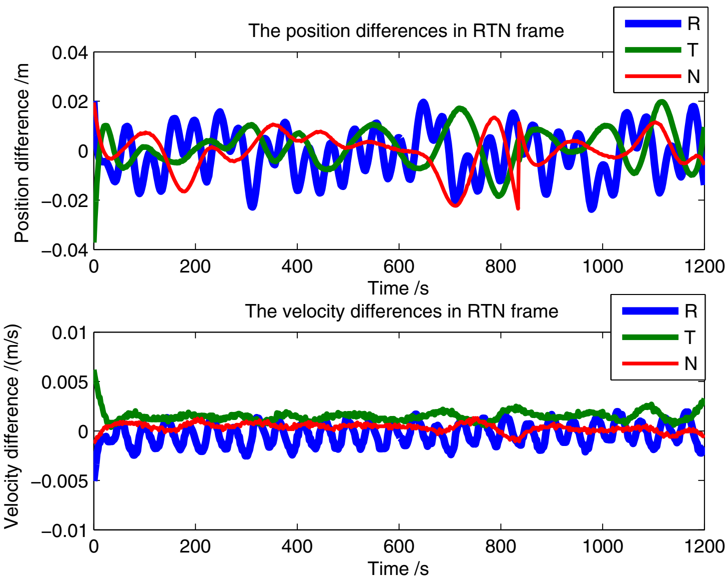

Fig. 1 The calculated orbit errors of a LEO satellite in the RTN frame ($P = 30$)

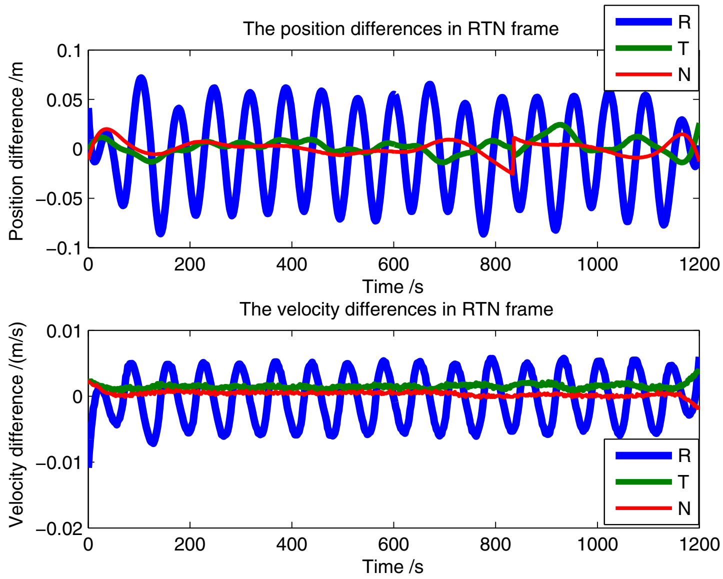

Fig. 2 The calculated orbit errors of a LEO satellite in the RTN frame ($P = 20$)

3.3 Example of Orbit Determination of A HEO Satellite

The former two examples are concerned with the orbits of small eccentricity, this example, however, will bring about the calculation for an orbit of greater eccentricity. The initial Kepler orbital elements of a MEO satellite at the epoch 2012-07-01 12:00:00 are: $a =$ 27907.356436 km, $e = 0.4$, $i = 75.967°$, $\Omega = 61.359°$, $\omega = 131.577°$, $M = 306.077°$. The noise and sampling rate are the same as those of former two examples. With $P = 24$, the orbit determination result is shown in Fig.5. Because the function approximation is proceeded directly on the position and velocity of the spacecraft, the results are almost unconcerned with the types of orbits. This example has conformed this conclusion.

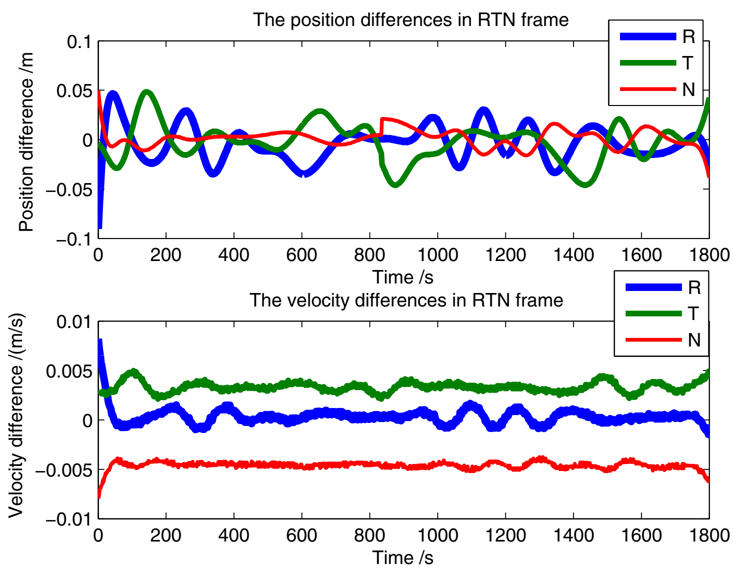

Fig. 3 The calculated orbit errors of a MEO satellite in the RTN frame ($P = 30$)

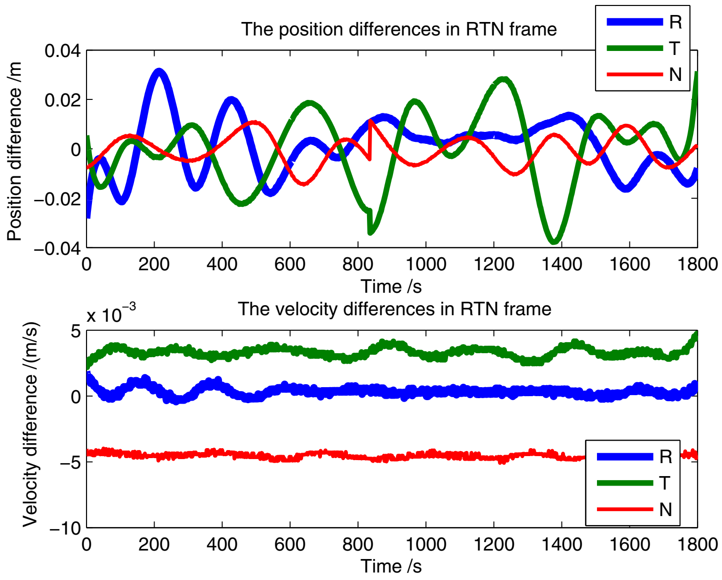

Fig. 4 The calculated orbit errors of a MEO satellite in the RTN frame ($P = 20$)

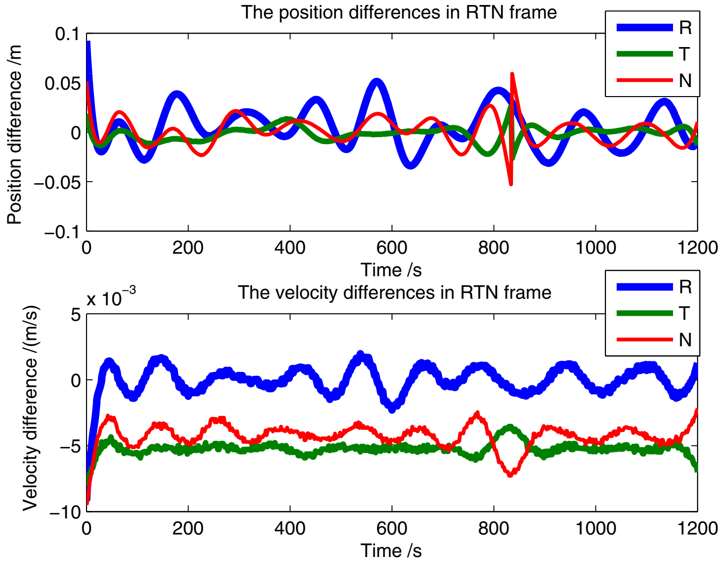

Fig. 5 The calculated orbit errors of a HEO satellite in the RTN frame ($P = 24$)

Up to now, the solutions of three representative orbits have been given. The results have conformed that the orbit determination of a spacecraft may be realized with the B-spline approximation method, which is verified to a certain extent to be effective.

4. CONCLUSIONS AND DISCUSSIONS

As analyzed from theory and calculation examples, the advantages of the B-spline orbit approximation are tentatively concluded as follows:

(1) In the case of relatively frequent orbit manoeuvre, or that it is unable to know when an orbit changing happens, the dynamical modeling can not be performed, but the orbit determination can be easily treated with the B-spline approximation method.

(2) The method is irrelevant to the choice of coordinate system. Taking the earth navigation satellite as an example, users show generally their concerns only for the ephemeris under the Earth-fixed coordinate system. The orbit determination may be directly made under the Earth-fixed coordinate system with the B-spline approximation method. Under different coordinate systems, the orbit configurations are the same, only the coefficients differ.

(3) The B-spline approximation method is applicable to the determination of both the circulatory orbit and transfer one. For instance, when a satellite plans reentry and return, in the phase from a low orbit to a high orbit, as well as in the transfer phase and reentry phase, the trajectory determination can be continuously carried out. Of course, in this case, it needs further investigation whether the B-spline approximation method presented in this article may meet the demands on the engineering precision and real-time treatment.

(4) This method can be verified and supplemented each other with the dynamical method.

(5) The direct calculation of the position and velocity of a spacecraft using the function approximation with neither the orbit integration of state quantities nor the calculation of transfer matrix leads to the significant simplification of orbit determination programming and to the substantial reduction in the amount of calculation. Evidently, the reduction of calculation is of significance, because of the relative shortage of calculation resources in the case of satellite-base orbit determination.

(6) The B-spline approximation method has no particular demands upon the preliminary orbit. If for some reasons there should be no preliminary orbit, it will be difficult to make an orbit determination with the dynamical method, but a rapid and direct orbit determination will be realized with the B-spline approximation method. A lot of orbit determination tests for earth satellites and lunar satellites demonstrate that even if all preliminary parameters are taken as zero, the solutions are always convergent.

There exist also some limitations and deficiencies with the method presented in this article:

(1) Since this method is non-dynamical one, the original significance of orbit dynamics has been lost, hence the method presented in this article can not be used in orbit extrapolation. This also means that although the use of mathematical skill is advantageous to the engineering or theory in certain conditions, it can not replace the study of orbit dynamics. This should be brought to notice.

(2) In this article, the data are of relatively simple type, which degenerates the partial differential matrix of the observational quantities relative to the state quantities into a unit matrix. In the practical work, it happens often that the observational quantities are not a linear combination of the state quantities, so it is required that the partial differential matrix of the observational quantities relative to the state quantities should be derived. In principle, there is no difficulty resulted by this process, and the discussion held in this article is not hindered.

By the way, in recent years thank of the development of measuring techniques, for some spacecraft may be obtained at a moment the relatively abundant observational data, with which a direct positioning may be carried out for the spacecraft, and its trajectory may be further fitted out. This process was designated by some researchers as kinetic orbit determination. There are, however, some basic differences between the method of this article and the above process. They are:

(1) To determine a kinetic trajectory, the spacecraft is positioned at the different points along its orbit, with the internal relation of the orbit itself neglected.

(2) To determine a kinetic trajectory, a sufficient amount of observational data is required at once, otherwise the positioning can not be realized, nor the determination of the final trajectory. This demand is difficult to satisfy in some cases, but the orbit determination of a spacecraft may be realized with the numerical approximation method presented in this article, even once a single ranging datum is got only.

In order to test its performance of orbit approximation, the B-spline method has been also applied to the orbit determination of the landing phase of a lunar probe, the orbit determinations are realized with the measured data of VLBI (Very Long Baseline Interferometry) and USB (Radar System of United S Band) types, respectively. All exhibit a rather good robustness.

In recent decades, the orbit determination methods remain almost unchanged; nevertheless, with the developing domestic and foreign space techniques and the diversifying needs and conditions of the measurement and control missions, many new problems start to crop up in succession. It is necessary to have appropriate methods to suit the diverse conditions. As this article is concerned, besides the spline, the nerve net, SVM (Support Vector Machine) system [14] and others are all the effective means of function approximation. They are expected to be introduced into the orbit determination, so that the problems difficult to treat with classic methods may be responded. A lot of work remains to do.

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